

Boundary Conditions, Supersymmetry and A -field Coupling for an Open String in a B -field Background

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ABSTRACT

We discuss the non-linear sigma model representing a NSR open string in a curved background with non-zero $B_{\mu\nu}$ -field. With this coupling the theory is not automatically supersymmetric, due to boundary contributions. When $B = 0$ supersymmetry is ensured by the conditions that follow as the boundary contribution to the field equations. We show that inclusion of a particular boundary term restores this state of affairs also in the presence of a B -field. The boundary conditions derived from the field equations in this case agree with those that have been proposed for constant B -field. A coupling to a boundary A_μ -field will modify both the boundary conditions and affect the supersymmetry. It is shown that there is an A -coupling with non-standard fermionic part that respects both the supersymmetry and the shift symmetry (in the B and A fields), modulo the (modified) boundary conditions.

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1 Introduction

Lately, interest in relevance of noncommutative geometry for string theory [1] has led to investigations of open strings propagating in a (constant) two-form background [2, 3]. In this context generalization of the periodic (antiperiodic) boundary condition for open string fermions (and the bosonic counterpart) have been discussed [4, 5, 3], but not derived from an action. In this paper we study the (globally) supersymmetric sigma model action representing an open NSR string in a curved background with non-zero B -field. In the most general setting, both the metric and the B -field are taken to be arbitrary background fields. With this coupling the theory is not automatically supersymmetric, due to boundary contributions. When $B = 0$, however, supersymmetry is ensured by the conditions that follow as the boundary contribution to the field equations. We show that inclusion of a particular boundary term restores this state of affairs also in the general case. The boundary conditions derived from the field equations in this case agree with those that have been discussed for constant B -field. We further consider the coupling to a boundary A_μ -field. Such a coupling will modify the boundary conditions and affect the supersymmetry, but is needed for invariance under the combined gauge transformation of the B -field and shift of the A -field. It turns out that there is a A -coupling with non-standard fermionic part that respects both the supersymmetry and the shift symmetry modulo the (modified) boundary conditions. A remarkable feature of this A -coupling on the boundary is that, whereas it is not supersymmetric by itself, the boundary conditions nevertheless ensures invariance under supersymmetry (of the whole action).

We emphasize that our attitude is to take seriously the boundary conditions derived from the total action, including background B field and boundary A -field, and try to reconcile them both with supersymmetry (and shift symmetry).

The paper is organized as follows: Section 2.1 deals with the known case of a constant B -field and a flat Minkowski metric. Here we rederive the known boundary conditions from an action by adding a boundary term to the standard Lagrangian. We also prepare the ground for the generalisation to a non-constant metric and B -field presented in Section 2.2. In Section 3 we give the boundary coupling of the A -field, the corresponding boundary conditions for the whole action and the proof of boundary supersymmetry. In Section 4, for completeness, we sketch the covariant quantization of the model with constant background fields, discuss the question of space-time symmetry, exhibit the breaking of the Lorentz group and construct the vertex operator for a massless boson. Section 5 contains our comments and conclusions.

2 Construction of the action

2.1 Constant B -field

In this subsection we present the construction of the globally supersymmetric world-sheet action for the open string in Minkowski space in a constant B -field. (The discussion in the literature has been rather inconclusive [2]-[5].) To display the ideas, the presentation will be very explicit. Exactly the same logic will be applied to the non-constant case in the next subsection.

Many questions can be dealt with without explicit knowledge of the correct action. A constant B -field will not modify the bulk physics, (the equations of motion and the stress tensor, e.g.), but only the boundary conditions. From the bosonic theory we know the correct boundary condition for the coordinate X^μ :

$$[E_{\nu\mu}\partial_+X^\nu - E_{\mu\nu}\partial_-X^\nu]|_{\sigma=0,\pi} = 0, \quad (2.1)$$

where $E_{\mu\nu} = \eta_{\mu\nu} + B_{\mu\nu}$. Using the supersymmetry transformations (see Appendix for notation)

$$\delta X^\mu = -\epsilon^+\psi_+^\mu - \epsilon^-\psi_-^\mu, \quad (2.2)$$

$$\delta\psi_+^\mu = -i\epsilon^+\partial_+X^\mu, \quad (2.3)$$

$$\delta\psi_-^\mu = -i\epsilon^-\partial_-X^\mu, \quad (2.4)$$

we find (2.1) to be the boundary supersymmetry transformation of an expression involving the fermions

$$\eta(\eta_{\mu\nu}X'^\nu - B_{\mu\nu}\dot{X}^\nu)|_{\sigma=0,\pi} = \frac{i}{2}\delta(E_{\nu\mu}\psi_+^\nu \mp E_{\mu\nu}\psi_-^\nu)|_{\sigma=0,\pi}, \quad (2.5)$$

where $\eta \equiv \epsilon^+ = \pm\epsilon^-$, $\eta_{\mu\nu}$ is the Minkowski metric. For the model to be supersymmetric, we thus have to require the expression on the right hand side to vanish. Our task is then to construct an action which gives rise to both (2.1) and the fermionic boundary condition implicit in (2.5). We start from the standard superfield action

$$S = \frac{1}{4\pi\alpha'} \int d^2\xi d^2\theta D_+\Phi^\mu D_-\Phi^\nu E_{\mu\nu}, \quad (2.6)$$

with constant $E_{\mu\nu}$ and manifest bulk supersymmetry. The component action that results from (2.6) is

$$\begin{aligned} S = & -\frac{1}{4\pi\alpha'} \int d^2\xi (\partial_\alpha X_\mu \partial^\alpha X^\mu + i\bar{\psi}^\mu \rho^\alpha \partial_\alpha \psi_\mu + \\ & + \epsilon^{\alpha\beta} B_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu + i\epsilon^{\alpha\beta} B_{\mu\nu} \bar{\psi}^\mu \rho_\alpha \partial_\beta \psi^\nu) \end{aligned} \quad (2.7)$$

where we have kept all boundary terms. Varying the action we find the usual field equations and the boundary contributions, which should vanish. We find that we must require

$$\left(\delta X^\mu [\partial_- X^\nu E_{\mu\nu} - \partial_+ X^\nu E_{\mu\nu}] + i[\psi_-^\mu \delta \psi_-^\nu E_{\nu\mu} - \psi_+^\mu \delta \psi_+^\nu E_{\mu\nu}] \right)_{|\sigma=0,\pi} = 0. \quad (2.8)$$

An additional condition follows from the the supersymmetry variation of the action (2.7), namely

$$\eta[\partial_+ X^\mu \psi_-^\nu E_{\mu\nu} \mp \psi_+^\mu \partial_- X^\nu E_{\mu\nu}]_{|\sigma=0,\pi} = 0. \quad (2.9)$$

Correct boundary conditions for X^μ and ψ^μ should ensure that both (2.8) and (2.9) are satisfied. The bosonic boundary condition (2.1) will cancel the bosonic variation in (2.8). Using it in (2.9) and trying to choose the fermionic boundary conditions such that both (2.8) and (2.9) are satisfied, however, we run into contradictions. In the presence of the B -field there is no such fermionic boundary condition⁴.

Without changing the bosonic part, the way forward is to add boundary terms, i.e., total derivatives, to the action (2.7). For constant B -field, there are two (essentially unique) $2D$ Lorentz-invariant boundary terms involving two fermions and one derivative. They give us the following additional action:

$$S_{bound} = -\frac{1}{4\pi\alpha'} \int d^2\xi \left[\alpha \left(\epsilon^{\alpha\beta} B_{\mu\nu} \bar{\psi}^\mu \rho_\alpha \partial_\beta \psi^\nu \right) + \beta \left(B_{\mu\nu} \bar{\psi}^\mu \rho^\alpha \partial_\alpha \psi^\nu \right) \right]. \quad (2.10)$$

Analysing the sum of (2.7) and (2.10) we find solutions to the relations corresponding to (2.8) and (2.9) for $\alpha = \beta = i$. Thus the boundary term (2.10) has the form

$$S_{bound} = -\frac{1}{4\pi\alpha'} \int d^2\xi (2i B_{\mu\nu} \psi_+^\mu \partial_- \psi_+^\nu) = -\frac{1}{4\pi\alpha'} \int d^2\xi (i \partial_- (B_{\mu\nu} \psi_+^\mu \psi_+^\nu)), \quad (2.11)$$

and the sum of the actions is

$$S = -\frac{1}{4\pi\alpha'} \int d^2\xi (\partial_\alpha X_\mu \partial^\alpha X^\mu + \epsilon^{\alpha\beta} B_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu + i E_{\nu\mu} \bar{\psi}^\mu \rho^\alpha \partial_\alpha \psi^\nu). \quad (2.12)$$

The boundary conditions are (2.1) and

$$(E_{\nu\mu} \psi_+^\nu \mp E_{\mu\nu} \psi_-^\nu)_{|\sigma=0,\pi}, \quad (2.13)$$

related as in (2.5). When these conditions are imposed, the action (2.12) is supersymmetric. Clearly, this action cannot be written in standard bulk-superfield form due to the boundary term.

⁴There is a trivial solution $\psi_-^\mu = \pm \psi_+^\mu = \text{const. spinor}$ which we are not interested in.

2.2 Non-constant metric and B -field

In this subsection we extend the previous analysis to include a general metric $g_{\mu\nu}$ and antisymmetric two-form $B_{\mu\nu}$. For ease of notation, we use superfield language at some of the steps where we used component notation in the previous subsection.

Again we start from the superfield version of the theory⁵

$$S = \int d^2\xi d^2\theta \mathcal{L} = \int d^2\xi d^2\theta D_+ \Phi^\mu D_- \Phi^\nu E_{\mu\nu}(\Phi), \quad (2.14)$$

where the θ independent part (denoted by $|$) of E is $E_{\mu\nu}| \equiv g_{\mu\nu} + B_{\mu\nu}$ with g and B the space-time metric and antisymmetric tensor field, respectively.

Using (A.52) of the appendix, we find the supersymmetry variation of a general action to be

$$\delta_\varepsilon S = -i \int d^2\xi \left\{ \varepsilon^+ \partial_+ D_- - \varepsilon^- \partial_- D_+ \right\} \mathcal{L}|. \quad (2.15)$$

For the special case of (2.14), (2.15) implies that

$$\eta \{ (D_- \pm D_+) D_+ \Phi^\mu D_- \Phi^\nu E_{\mu\nu}(\Phi) \} | \quad (2.16)$$

has to vanish at $\sigma = 0, \pi$. (Here $\eta \equiv \varepsilon^+ = \pm \varepsilon^-$, as before).

The field equations for the action with Lagrangian (2.14) are obtained from the general variation

$$\begin{aligned} \delta S &= \int d^2\xi d^2\theta \left[\delta\Phi \left\{ (D_+ D_- \Phi^\nu)(E_{[\nu\mu]} + D_+ \Phi^\rho D_- \Phi^\nu (E_{\rho[\nu\mu]} - E_{\mu\nu\rho})) \right\} \right] \\ &\quad + \int d^2\xi d^2\theta [D_+ \{ \delta\Phi^\mu D_- \Phi^\nu E_{\mu\nu} \} - D_- \{ D_+ \Phi^\mu \delta\Phi^\nu E_{\mu\nu} \}] \\ &\equiv I_1 + I_2. \end{aligned} \quad (2.17)$$

Here I_1 gives the field equations in the bulk (of the world sheet) and I_2 is a boundary term which implies the vanishing of

$$[D_- \{ \delta\Phi^\mu D_- \Phi^\nu E_{\mu\nu} \} - D_+ \{ D_+ \Phi^\mu \delta\Phi^\nu E_{\mu\nu} \}] | \quad (2.18)$$

at $\sigma = 0, \pi$.

As before, it is easy to convince oneself that the two requirements (2.16) and (2.18) are incompatible except when $B_{\mu\nu} = 0$. Since the discrepancy is purely at the boundary of the world sheet we want to add a boundary term. Guided by a study of the case of constant $E_{\mu\nu}$ (see equation (2.11)), we add the term

$$\mathcal{L}_B = -i \partial_- (\psi_+^\mu \psi_+^\nu B_{\mu\nu}(X)) \quad (2.19)$$

⁵In this section we use the normalization $\alpha' = (4\pi)^{-1}$.

to the component Lagrange density. Adding the contributions from (2.19) to (2.18), the boundary term (in components) reads

$$\begin{aligned}
& - \left\{ i \left[\delta\psi_-^\mu \psi_-^\nu - \delta\psi_+^\mu \psi_+^\nu \right] E_{\mu\nu} \right. \\
& + \left. \delta X^\mu \left[\partial_- X^\nu E_{\mu\nu} - \partial_+ X^\nu E_{\nu\mu} + i\psi_-^\nu \psi_-^\rho E_{\mu\nu,\rho} - i\psi_+^\nu \psi_+^\rho E_{\nu[\mu,\rho]} \right] \right\}_{\sigma=0,\pi} = 0. \quad (2.20)
\end{aligned}$$

(Comma denotes a partial derivative.) Our solution for the present case is obtained starting from formally the same fermionic condition as in the constant case, i.e., from (2.13) with non-constant $E_{\mu\nu}$. Substituting the ψ variations (at the boundary) that result from (2.13) into (2.20) we find that (2.1) gets replaced by

$$\left[i \{ \partial_+ X^\nu E_{\nu\mu} - \partial_- X^\nu E_{\mu\nu} \} \pm \psi_-^\rho \psi_+^\sigma E_{\sigma\rho,\mu} + \psi_-^\sigma \psi_-^\rho E_{\mu\sigma,\rho} - \psi_+^\sigma \psi_+^\rho E_{\sigma\mu,\rho} \right]_{\sigma=0,\pi} = 0 \quad (2.21)$$

The relations (2.13) and (2.21) in conjunction with the F -field equations are sufficient to show that the sum of the boundary supersymmetry variation (2.16) and the variation of (2.19) vanish. The auxiliary F -field equation follows from I_1 in (2.17) and reads

$$2F_{+-}^\nu g_{\mu\nu} + \psi_+^\rho \psi_-^\nu (E_{\mu\nu,\rho} + E_{\rho\mu,\nu} - E_{\rho\nu,\mu}) = 0. \quad (2.22)$$

We note for future reference that it contains the B -field as a field-strength only. One can check that the boundary supersymmetry variation of (2.13) is proportional to (2.21), the same relation as in (2.5). In checking this one should keep in mind that the supersymmetry transformations look rather involved due to (2.22).

In the constant case there are only two fermionic boundary terms possible. However, in the nonconstant case there are infinite many fermionic boundary terms available (for instance, $B_{\mu\nu,\rho} \psi_+^\mu \psi_+^\nu \psi_+^\rho$). All these terms (except the terms in (2.10)) contain derivatives of the background fields.

3 Matter coupling on the boundary

In this section we discuss the possible coupling to an A_μ -field on the boundary. We argue that there is an essentially unique such coupling that preserves supersymmetry and the shift symmetry (defined below).

The bosonic sigma model with a B -field coupling is invariant up to boundary terms under the transformation

$$\delta B_{\mu\nu} = \partial_{[\mu} \Lambda_{\nu]}. \quad (3.23)$$

When a boundary is present the resulting boundary term is compensated by a shift $\delta A_\mu = \Lambda_\mu$ of an A -field on the boundary whose action is

$$S_A = \int d\tau A_\mu \dot{X}^\mu. \quad (3.24)$$

When we consider the supersymmetric sigma model, two problems confront us: The addition of an A -action will change the boundary conditions and we must preserve supersymmetry. Remarkably, both these can be resolved.

We start from the supersymmetric action of the previous section, i.e., (2.14) with the addition of (2.19). We then observe that a field redefinition $B_{\mu\nu} \rightarrow B_{\mu\nu} + F_{\mu\nu} \equiv \hat{B}_{\mu\nu}$ will only give a contribution on the boundary, due to the invariance (3.23). We collect all the A -terms to a boundary action

$$S_A = \int d\tau \left(-2A_\mu \dot{X}^\mu + \frac{i}{2}(\psi_+^\mu + \psi_-^\mu) F_{\mu\nu} (\psi_+^\nu - \psi_-^\nu) \right). \quad (3.25)$$

This is not the usual supersymmetrization of (3.24) (see (4.46) below); one has to keep in mind that the supersymmetry only holds modulo the boundary conditions. These now read

$$\left(\hat{E}_{\nu\mu} \psi_+^\nu \mp \hat{E}_{\mu\nu} \psi_-^\nu \right) |_{\sigma=0,\pi}, \quad (3.26)$$

and

$$\left[i \left\{ \partial_+ X^\nu \hat{E}_{\nu\mu} - \partial_- X^\nu \hat{E}_{\mu\nu} \right\} \pm \psi_-^\rho \psi_+^\sigma \hat{E}_{\sigma\rho,\mu} + \psi_-^\sigma \psi_-^\rho \hat{E}_{\mu\sigma,\rho} - \psi_+^\sigma \psi_+^\rho \hat{E}_{\sigma\mu,\rho} \right]_{\sigma=0,\pi} = 0, \quad (3.27)$$

where \hat{E} contains B and F in the combination \hat{B} . Note that this implies that the boundary conditions are invariant under the shift symmetry. Note also that the auxiliary F_{+-}^μ field equations that we need in the supersymmetry check are invariant too (see the comment after (2.22)). In fact, since our present model is related to the supersymmetric one by a shift invariant field redefinition, it is supersymmetric and shift-invariant by construction. This has also been explicitly verified.

Some comments are in order. First, we again stress that the field-redefinition of B is a tool which allows us to identify the shift-invariant action. Second, superficially, (3.25) depends on both $\psi_+ + \psi_-$ and $\psi_+ - \psi_-$. However, using (3.26) we may eliminate one in favour of the other. Third, one may ask what happens to our model in the limit of vanishing B -field and the relation to the standard supersymmetric A -field action as given in (4.46) below. When $B = 0$ there are still F -contributions to (3.26-3.27), and they do indeed ensure supersymmetry (modulo these conditions). The usual A -field coupling, on the other hand, will give contributions to the boundary conditions that are in fact incompatible with supersymmetry of the full action.

4 Covariant quantization

In this section we would for completeness like to sketch the covariant quantization of the model and specifically discuss the issue of the broken Lorentz group. At the end the con-

struction of the vertex operator for emission of massless boson is given and some problems related to the shift symmetry are pointed out.

Let us look at the theory with constant $B_{\mu\nu}$ as given by (2.12), with Neumann boundary conditions in all directions. Since the equations of motion are not modified we can solve them in the standard way. The presence of a constant B -field results in the boundary conditions which relate the left and right movers in a non-trivial way [2]. The general solution of the bosonic equation of motion, satisfying the bosonic boundary condition, has the form

$$X^\mu(\tau, \sigma) = q^\mu + 2\alpha'(G^{-1})^{\mu\nu} p_\nu \tau - 2\alpha'\theta^{\mu\nu} p_\nu \sigma + \sqrt{2\alpha'} \sum_{n \neq 0} \frac{e^{-in\tau}}{n} (i\alpha_n^\mu \cos n\sigma + B^\mu{}_\nu \alpha_n^\nu \sin n\sigma), \quad (4.28)$$

where we use the notation (for details see [6])

$$G_{\mu\nu} \equiv E_{\rho\mu} \eta^{\rho\sigma} E_{\sigma\nu} = \eta_{\mu\nu} - B_{\mu\rho} \eta^{\rho\sigma} B_{\sigma\nu}, \quad \theta^{\mu\nu} \equiv -B^\mu{}_\sigma (G^{-1})^{\sigma\nu}. \quad (4.29)$$

We should also solve the equations of motion for the fermionic coordinates

$$\partial_- \psi_+^\nu = 0, \quad \partial_+ \psi_-^\nu = 0, \quad (4.30)$$

taking into account the fermionic boundary conditions (2.13)

$$E_{\nu\mu} \psi_+^\nu \mp E_{\mu\nu} \psi_-^\nu|_{\sigma=0,\pi} = 0, \quad (4.31)$$

where the plus sign corresponds to Neveu-Schwarz (NS) and the minus to Ramond (R) conditions. As usual, the overall relative sign is conventional, so without loss of generality we set $E_{\nu\mu} \psi_+^\nu(\tau, 0) = E_{\mu\nu} \psi_-^\nu(\tau, 0)$. We thus have the following solutions of (4.30) and (4.31):

$$\psi_-^\mu = \eta^{\mu\nu} E_{\rho\nu} \frac{1}{\sqrt{2}} \sum_{r \in Z+1/2} b_r^\rho e^{ir(\tau-\sigma)} \quad (4.32)$$

$$\psi_+^\mu = \eta^{\mu\nu} E_{\nu\rho} \frac{1}{\sqrt{2}} \sum_{r \in Z+1/2} b_r^\rho e^{ir(\tau+\sigma)} \quad (4.33)$$

for the NS sector and

$$\psi_-^\mu = \eta^{\mu\nu} E_{\rho\nu} \frac{1}{\sqrt{2}} \sum_{n \in Z} d_n^\rho e^{in(\tau-\sigma)} \quad (4.34)$$

$$\psi_+^\mu = \eta^{\mu\nu} E_{\nu\rho} \frac{1}{\sqrt{2}} \sum_{n \in Z} d_n^\rho e^{in(\tau+\sigma)} \quad (4.35)$$

for the R sector. The standard anticommutation relations for the fermionic coordinates

$$\{\psi_A^\mu(\tau, \sigma), \psi_B^\nu(\tau, \sigma')\} = \pi \eta^{\mu\nu} \delta(\sigma - \sigma') \delta_{AB} \quad (4.36)$$

imply anticommutation relations for the modes

$$\{b_r^\mu, b_s^\nu\} = (G^{-1})^{\mu\nu} \delta_{r+s}, \quad \{d_n^\mu, d_m^\nu\} = (G^{-1})^{\mu\nu} \delta_{n+m} \quad (4.37)$$

The canonical commutation relations for the bosonic counterpart imply the following commutator relations

$$[\alpha_n^\mu, \alpha_m^\nu] = n\delta_{n+m}(G^{-1})^{\mu\nu}, \quad [q^\mu, p_\nu] = i\delta_\nu^\mu, \quad [q^\mu, q^\nu] = 2\pi i\alpha'\theta^{\mu\nu}. \quad (4.38)$$

The super Virasoro generators are

$$L_n = \frac{1}{2} \sum_{m=-\infty}^{\infty} \alpha_{-m}^\mu G_{\mu\nu} \alpha_{m+n}^\nu + \frac{1}{2} \sum_{r=-\infty}^{\infty} (r + \frac{1}{2}n) b_{-r}^\mu G_{\mu\nu} b_{n+r}^\nu \quad (NS) \quad (4.39)$$

$$L_n = \frac{1}{2} \sum_{m=-\infty}^{\infty} \alpha_{-m}^\mu G_{\mu\nu} \alpha_{m+n}^\nu + \frac{1}{2} \sum_{m=-\infty}^{\infty} (m + \frac{1}{2}n) d_{-m}^\mu G_{\mu\nu} d_{m+n}^\nu \quad (R) \quad (4.40)$$

$$G_r = \sum_{n=-\infty}^{\infty} \alpha_{-n}^\mu G_{\mu\nu} b_{r+n}^\nu \quad (NS) \quad (4.41)$$

$$F_n = \sum_{m=-\infty}^{\infty} \alpha_{-m}^\mu G_{\mu\nu} d_{n+m}^\nu \quad (R), \quad (4.42)$$

where normal ordering is assumed in all expressions. These generators give the standard super Virasoro algebra with central extension. The B -field does not change the anomaly in the super Virasoro algebra and the system can thus be quantized as usual.

The presence of a B -field breaks the Lorentz symmetry to the $SO(2r-1, 1) \otimes (SO(2))^{d/2-r}$ subgroup where $(d-2r)$ is the rank of the matrix $B_{\mu\nu}$. Let us take a look at the spectrum. As in the bosonic case, to avoid trouble we should define the mass using the new metric $G_{\mu\nu}$. The NS sector is the same as usual. The ground state corresponds to a tachyon with mass $M^2 = -p_\mu (G^{-1})^{\mu\nu} p_\nu = -1/(2\alpha')$. The state $\zeta_\mu b_{-1/2}^\mu |0, k\rangle$ is a massless vector with respect to the new Lorentz group. In the R sector we have a fermionic zero mode that makes the R ground state degenerate, since $[d_0^\mu, L_0] = 0$.

$$\{d_0^\mu, d_0^\nu\} = (G^{-1})^{\mu\nu}. \quad (4.43)$$

Thus the R ground state transforms as a space-time fermion under the new Lorentz group. The zero modes are given by

$$d_0^\mu = \frac{1}{\sqrt{2}} \left(\frac{1}{\eta - B} \right)^{\mu\nu} \Gamma_\nu \quad (4.44)$$

where Γ_ν are the standard gamma matrices as in [7]. One can interpret this to mean that as the B -field varies from 0 to ∞ there is smooth interpolation between Lorentz symmetry and the R-symmetry.

We may use the above results to construct the vertex operator for emission of massless boson $\zeta_\mu b_{-1/2}^\mu |0, k\rangle$ along the standard lines (see discussion in section 4.2.3 i volume 1 of [7])

using the modified commutation relations (4.37), (4.38) and the super Virasoro generators (4.39)-(4.42). The result is

$$V = (\zeta_\mu \dot{X}^\mu(0) - \zeta_\mu \Psi^\mu(0) k_\nu \Psi^\nu(0)) e^{ik_\mu X^\mu(0)} \quad (4.45)$$

where $\Psi^\mu(0) = 1/2(\psi_+^\mu(0) + \psi_-^\mu(0))$ and ζ_μ is the polarization vector of the spin-one field. Note that (4.45) is identical to that discussed in [3]. If we naively read off the matter coupling to the sigma model [8], we expect it to be

$$S_A = \int d\tau \left(A_\mu \dot{X}^\mu - \frac{1}{2i} F_{\mu\nu} \Psi^\mu \Psi^\nu \right). \quad (4.46)$$

This concides with the boundary interactions discussed in [3, 9, 10] but disagrees with (3.25). The difference seems to emanate from our different approaches. The interaction (4.46) is supersymmetric by itself, independent of the boundary conditions. The interaction (3.25) on the other hand, is only supersymmetric together with the rest of the action and with the appropriate boundary conditions (derived from the total action) imposed. Note also that the full action with (3.25) included respects the shift symmetry whereas we do not know how to realize the shift symmetry in a supersymmetric way with the interaction in (4.46).

5 Discussion

We first make some comment on the boundary conditions that we have derived. Let us start from the case when $B_{\mu\nu} = 0$ and $g_{\mu\nu}$ is arbitrary. In this situation the boundary conditions (2.13) for the fermions are

$$\{\psi_+^\mu \mp \psi_-^\mu\}_{\sigma=0,\pi} = 0. \quad (5.47)$$

The F-field equation (2.22) has the form

$$F_{+-}^\mu + \psi_+^\rho \psi_-^\nu \Gamma_{\nu\rho}^\mu = 0, \quad (5.48)$$

where $\Gamma_{\nu\rho}^\mu$ are the Christoffel symbols. Thus on the boundary the equation (5.48) reduces to $F_{+-}^\mu = 0$ because of the symmetries of the Christoffel symbols and the boundary conditions (5.47). Therefore the supersymmetric transformation restricted to the boundary is exactly the same as in the constant case (2.2). Further, the boundary condition (2.21) collapses to $X'^\mu = 0$. We thus see that the curved metric by itself does not make the boundary conditions more complicated than in the constant case. However some problems might arise when we try to introduce Dirichlet conditions in some of the directions. Like in the case with constant B -field, discussed below, the mixed components of the metric g_{im} are the source of these difficulties. (Here i is Dirichlet and m is Neumann directions.)

In our discussion so far we have used Neumann boundary conditions⁶ in all directions. Thus we had in mind, e.g., open strings that represent fluctuations of a D9-brane. Restricting to constant E case, we may ask which other boundary conditions the action (2.12) admits. In other words we take a look at a general Dp-brane with $p < 9$. We assume no special direction for the background $B_{\mu\nu}$ -field⁷. To define the Dp-brane we impose the following Neumann conditions

$$\partial_- X^n E_{mn} - \partial_+ X^n E_{nm}|_{\sigma=0,\pi} = 0, \quad E_{nm} \psi_+^n \mp E_{mn} \psi_-^n|_{\sigma=0,\pi} = 0, \quad n, m = 0, 1, \dots, p \quad (5.49)$$

and the Dirichlet conditions

$$\dot{X}^i = 0|_{\sigma=0,\pi}, \quad \psi_+^i \pm \psi_-^i|_{\sigma=0,\pi} = 0, \quad i = p+1, \dots, 9. \quad (5.50)$$

In (5.49) and (5.50) the bosonic and fermionic conditions are related to each other through the supersymmetry transformations (2.2). By plugging these conditions into the corresponding variations of the action (2.12) one will find that in the generic situation there are non-vanishing terms proportional to the mixed component B_{im} .

One may view this slightly differently: If we impose the Dirichlet conditions (5.50) in some directions and try to choose other boundary conditions to cancel the corresponding variations then in addition to (5.49) and (5.50) we would have to introduce boundary conditions which mix the Neumann and Dirichlet directions. A theory with such boundary conditions would be inconsistent, as may be easily seen. However, by a meticulous choice of the Dirichlet directions we may obtain that $B_{im} = 0$ and then things will work out. We may thus interpret this as restrictions on the orientation of those Dp-branes for which the action (2.12) is consistent. E.g., there is no problem if the B -field is non-zero only along the brane. The restrictions have a natural interpretation related to the broken Lorentz group.

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⁶By Neumann boundary conditions we mean conditions that ensure that there is no flow of momentum off the endpoints of the string, not to be confused with the mathematical notion of Neumann boundary conditions.

⁷One may view this as embedding a Dp-brane in a D9-brane. We shall see that in the presence of a B -field this cannot be arbitrarily done.

A Appendix

Throughout the paper we use μ, ν, \dots as spacetime indices. The two dimensional spinor indices are $(a, b, \dots = 0, 1)$ and $(\alpha, \beta, \dots = +, -)$ denote world sheet indices. We also use superspace conventions where the spinor coordinates are labeled θ^\pm and the covariant derivatives D_\pm and supersymmetry generators Q_\pm satisfy

$$\begin{aligned} D_+^2 &= i\partial_+, & D_-^2 &= i\partial_-, & \{D_+, D_-\} &= 0 \\ Q_\pm &= -D_\pm + 2i\theta^\pm \partial_\pm \end{aligned} \quad (\text{A.51})$$

where $\partial_\pm = \partial_0 \pm \partial_1$. In terms of the covariant derivatives, a supersymmetry transformation of a superfield Φ is then given by

$$\begin{aligned} \delta\Phi &\equiv (\varepsilon^+ Q_+ + \varepsilon^- Q_-)\Phi \\ &= -(\varepsilon^+ D_+ + \varepsilon^- D_-)\Phi + 2i(\varepsilon^+ \theta^+ \partial_+ + \varepsilon^- \theta^- \partial_-)\Phi \end{aligned} \quad (\text{A.52})$$

The components of a superfield Φ are defined via projections as follows:

$$\Phi| \equiv X, \quad D_\pm \Phi| \equiv \psi_\pm, \quad D_+ D_- \Phi| \equiv F_{+-}, \quad (\text{A.53})$$

where a vertical bar denotes “the $\theta = 0$ part of”. Choosing the world-sheet γ -matrices as

$$\rho^0 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad \rho^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \rho^3 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \quad (\text{A.54})$$

where $\{\rho^\alpha, \rho^\beta\} = +2\eta^{\alpha\beta}$ and $\eta^{\alpha\beta} = (-, +)$, the Majorana spinors ψ can be decomposed into two components with different chirality

$$\psi^\mp = \frac{1 \pm \rho^3}{2} \psi, \quad \rho^3 = \rho^0 \rho^1. \quad (\text{A.55})$$

Thus the spinors ψ are

$$\psi = \begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix} \quad (\text{A.56})$$

and $\bar{\psi} \equiv (\psi_+, \psi_-) = \psi^\dagger \rho^0$.

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